3D hierarchical interface-enriched finite element method: Implementation and applications

Soheil Soghrati\textsuperscript{a,b,c,*}, Hossein Ahmadian\textsuperscript{a}

\textsuperscript{a} Department of Mechanical and Aerospace Engineering, The Ohio State University, USA
\textsuperscript{b} Department of Materials Science and Engineering, The Ohio State University, USA
\textsuperscript{c} Simulation Innovation and Modeling Center, Columbus, OH, USA

\begin{abstract}
A hierarchical interface-enriched finite element method (HIFEM) is proposed for the mesh-independent treatment of 3D problems with intricate morphologies. The HIFEM implements a recursive algorithm for creating enrichment functions that capture gradient discontinuities in nonconforming finite elements cut by arbitrary number and configuration of materials interfaces. The method enables the mesh-independent simulation of multiphase problems with materials interfaces that are in close proximity or contact while providing a straightforward general approach for evaluating the enrichments. In this manuscript, we present a detailed discussion on the implementation issues and required computational geometry considerations associated with the HIFEM approximation of thermal and mechanical responses of 3D problems. A convergence study is provided to investigate the accuracy and convergence rate of the HIFEM and compare them with standard FEM benchmark solutions. We will also demonstrate the application of this mesh-independent method for simulating the thermal and mechanical responses of two composite materials systems with complex microstructures.
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1. Introduction

The standard finite element method (FEM) is one of the most reliable computational techniques for the numerical treatment of engineering problems and physical phenomena across different length scales [1–3]. While this method has been adopted as the underlying computational engine in a variety of commercial software programs and widely used for the analysis and computational design of engineering problems, its efficient implementation for simulating problems with complex and/or evolving morphologies remains a challenge [4–6]. The main roots of this challenge are (i) creating a realistic geometrical model of the problem and (ii) generating a conforming finite element (FE) mesh to discretize this virtual model. The time-consuming and laborious nature of performing these tasks, which often require considerable human intervention to ensure the construction of an appropriate FE model [7], can either considerably slow down the modeling process or lead to the construction of oversimplified models that ignore important morphological features. Therefore, the additional labor cost imposed by such limitations can impede the efficient application of the standard FEM for simulating problems such as composites [8], heterogeneous materials systems [9], and complex biomaterials [10,11].
Several numerical techniques including meshfree methods (MMs) [12,13] and mesh-independent FEMs [14,15] have been introduced to overcome the above mentioned restrictions of the standard FEM for simulating problems with intricate morphologies. For example, in the element-free Galerkin method (EFGM), the solution field is approximated using nodal data and their weight functions over a domain of influence [16,17]. The extended/generalized finite element method (XFEM/GFEM) [18–21] is a mesh-independent technique that has gained extensive attention for simulating problems with strong (field) and weak (gradient) discontinuities. This method implements the partition of unity scheme to create appropriate enrichment functions for reconstructing discontinuous phenomena in nonconforming finite elements. Thus, XFEM/GFEM eliminates the need to geometrically model materials interfaces or propagating cracks and allows the implementation of nonconforming FE meshes for discretizing the problem.

Recently, Soghrati et al. [22,23] introduced the interface-enriched generalized finite element method (IGFEM) for the semi-mesh-independent modeling and simulation of multiphase problems. The IGFEM enrichments are attached to new degrees of freedom (dofs) added to the original nonconforming FE mesh at the intersection point of materials interfaces with elements edges to capture weak discontinuities. The IGFEM application for simulating varying engineering problems with complex geometries are presented in [24–26]. A NURBS-based IGFEM has also been introduced in [27], which uses Non-Uniform Rational B-Splines (NURBS) shape functions for evaluating enrichment functions. However, a major limitation of the IGFEM that hinders its implementation for modeling multiphase problems with materials interfaces that are in a close proximity or contact is the difficulty associated with computing enrichment functions in elements cut by multiple materials interfaces. This difficulty emanates from the complex process involved in creating children sub-elements needed for both evaluating enrichments and performing the numerical integration.

To eliminate the above restriction of the IGFEM, Soghrati [28] introduced the hierarchical interface-enriched finite element method (HIFEM) for the fully mesh-independent modeling of 2D problems with intricate geometries. The HIFEM employs a recursive algorithm for constructing enrichment functions in elements intersecting with multiple materials interfaces and thus provides an automated approach for simulating the problem. These enrichments are hierarchically added to the approximate field to simulate weak discontinuities along materials interfaces, which can easily handle any number and configuration of materials phases inside the nonconforming element. One of the unique advantages of the HIFEM for modeling problems with highly complex morphologies such as heterogeneous composites is that different materials interfaces do not need to be aware of the existence of one another throughout the modeling process, which considerably facilitates the computer implementations and allows evaluating the enrichment functions associated with each interface independently.

In the current manuscript, we expand the HIFEM to 3D problems and in particular discuss varying computational geometry aspects associated with the construction of children elements in four-node nonconforming tetrahedral elements. The outline of the remainder of this manuscript is as follows: Section 2 presents the problem formulation together with a brief overview of the 2D HIFEM formulation. In Section 3, we present the 3D HIFEM algorithm and discuss required considerations for its computer implementation. A convergence study is provided in Section 4 followed by demonstrating the application of the 3D HIFEM for simulating the thermal and mechanical behavior of two composite materials systems representative volume elements (RVEs) with complex geometries.

2. Problem formulation and HIFEM algorithm

2.1. Governing equations

Consider an open domain $\Omega \subset \mathbb{R}^3$ composed of $m$ non-overlapping partitions $\Omega = \Omega_1 \cup \Omega_2 \ldots \cup \Omega_m$ representing $m$ different materials phases. The domain boundary $\partial \Omega = \Gamma_u \cup \Gamma_q$ has an outward unit normal vector $\mathbf{n}$ and is divided into two distinct regions ($\Gamma_u \cap \Gamma_q = \emptyset$) corresponding to Dirichlet and Neumann boundary conditions (BCs), respectively. The strong form of the steady-state conductive heat transfer governing equations in each subdomain is written as: Find the temperature field $u$ such that

$$
\begin{align*}
\kappa_i &\nabla^2 u = 0 \quad \text{in } \Omega_i \\
-\kappa_i &\nabla u \cdot \mathbf{n} = q \quad \text{on } \Gamma_q \\
u &= \bar{u} \quad \text{on } \Gamma_u,
\end{align*}
$$

where $\kappa_i$ is the thermal conductivity of the $i$th material phase, $\bar{u}$ is the prescribed temperature, and $q$ is the applied heat flux.

The strong form of the linear elasticity governing equations in $\Omega_i$ is expressed as: Find the displacement field $\mathbf{u}$

$$
\begin{align*}
\nabla \sigma + \mathbf{b} &= \mathbf{0} \quad \text{in } \Omega_i \\
\sigma &= C_i : \varepsilon \quad \text{in } \Omega_i \\
\varepsilon &= \frac{1}{2} \left( \nabla \mathbf{u} + \nabla \mathbf{u}^T \right) \quad \text{in } \Omega_i \\
\mathbf{u} &= \mathbf{u} \quad \text{on } \Gamma_u \\
\sigma \cdot \mathbf{n} &= \mathbf{t} \quad \text{on } \Gamma_t,
\end{align*}
$$

$$
where $\mathbf{b} : \Omega \rightarrow \mathbb{R}^3$ is the body force, $\mathbf{t} : \Gamma_1 \rightarrow \mathbb{R}^3$ is the applied traction, $\mathbf{u} : \Gamma_u \rightarrow \mathbb{R}^3$ is the prescribed displacement, and $C_i$ is the fourth-order stiffness tensor associated with the material phase $i$. Components of $C_i$ relate the Cauchy stress tensor $\sigma$ to the linearized strain tensor $\varepsilon$ as follows

$$\sigma = \lambda \text{tr}(\varepsilon) \mathbf{I} + 2\mu \varepsilon,$$

(3)

where $\lambda$ and $\mu$ are the Lame constants, $\text{tr}()$ is the trace operator, and $\mathbf{I}$ is the identity tensor.

Assuming that the temperature field $\psi$ is decomposed into $\psi = \psi_0 + \psi_d$, where $\psi_d|\Gamma_0 = \bar{\psi}$, the weak form of (1) is expressed as: Find $u_0 \in V := \{u_{0i} : \Omega \rightarrow \mathbb{R}, u_{0i}|\Gamma_0 = 0\}$ such that

$$\sum_{i=1}^m \int_{\Omega} \nabla(u_{0i} + u_d) \cdot \kappa_i \nabla \psi \, d\Omega + \int_{\Gamma_0} v Q \, d\Omega + \int_{\Gamma_0} v q d\Gamma = 0 \quad \forall \psi \in V.$$  

(4)

Similarly, decomposing the displacement field $\mathbf{u}$ into $\mathbf{u} = \mathbf{u}_0 + \mathbf{u}_d$ such that $\mathbf{u}_d|\Gamma_0 = \bar{\mathbf{u}}$, the weak form of (2) can be written as: Find $\mathbf{u}_0 \in W := \{\mathbf{u}_{0i} : \Omega \rightarrow \mathbb{R}^2, \mathbf{u}_{0i}|\Gamma_0 = \mathbf{0}\}$ such that

$$\sum_{i=1}^m \int_{\Omega} \mathbf{L}(u_{0i} + u_d) \cdot \mathbf{C} \mathbf{L}^T \mathbf{w} \, d\Omega + \int_{\Omega} \mathbf{w} \mathbf{b} \, d\Omega + \int_{\Gamma_0} \mathbf{w} \mathbf{t} d\Gamma = \mathbf{0} \quad \forall \mathbf{w} \in W,$$

(5)

where the differential operator $\mathbf{L}$ is given by

$$\mathbf{L} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial y} \\ 0 & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \end{bmatrix}.$$  

(6)

The Galerkin approximations $u_{0i}$ and $\mathbf{u}_{0i}$ of (4) and (5), respectively, can be evaluated by replacing $V$ and $W$ with proper finite dimensional spaces $V^h \subset V$ and $W^h \subset W$. In the context of the standard FEM, $V^h$ and $W^h$ are associated with the space of Lagrangian shape functions used to approximate the field in each element.

### 2.2. HIFEM formulation

This section provides a brief overview of the HIFEM formulation presented in [28] for simulating 2D problems with weak discontinuities. Since this formulation is identical for approximating both the conductive heat transfer and linear elasticity problems, we only focus on the former case. Discretizing the domain $\Omega \cong \Omega^k$ into $k$ nonconforming finite elements, the HIFEM approximation of the temperature field $u^h$ in (4) is given by

$$u^h = \sum_{i=1}^n N_i u_i + \sum_{j=1}^{n_{en}} \alpha_j^{(h)} \psi^j (h),$$

(7)

where $N_i$ is a set of $n$ Lagrangian shape function, $\psi^j (h)$ is a set of $n_{en}$ enrichment functions, $u_i$ is the temperature at node $i$ of the FE mesh, and $\alpha_j^{(h)}$ is the $j$th generalized dof created at the intersection point of the $h$th materials interface with the nonconforming element edges. The enrichment incorporated in the second term of (7) enables reconstructing weak discontinuities emanating from mismatch between material properties in the nonconforming elements of the background mesh.

The HIFEM enrichment function associated with each materials interface is computed independent of other interfaces using a recursive algorithm. This hierarchical procedure begins with assigning a level of hierarchy to materials interfaces intersecting with edges of nonconforming (parent) elements, which is evaluated based on the order of visiting each materials interface. The next step is to discretize the parent element into several children (integration) elements with respect to materials interfaces belonging to the first (highest) level of hierarchy. These children elements will serve as parent element for interfaces at lower hierarchical levels, enabling a recursive algorithm for the construction of children elements. This process is schematically shown in Fig. 1 for a three-node triangular element cut by two materials interfaces. The enrichment function $\psi^h_j$ associated with the $j$th interface node at the $h$th level of hierarchy is then computed as

$$\psi^h_j = \sum_{h=1}^{n_h} \sum_{k=1}^{n_{ch}^{(h)}} N^{(h)}_{j,k},$$

(8)

where $n_{ch}^{(h)}$ is the number of children elements connected to node $j$, $N^{(h)}_{j,k}$ is the $r$th Lagrangian shape function of the $k$th child element, and $n_h$ is the number of hierarchical levels.

The scaling factor $\psi^h_j$ is incorporated in (7) to avoid the formation of an ill-conditioned stiffness matrix due to the presence of children elements with high aspect ratios or excessively large angles. The scaling factor associated with the $j$th enriched node of a child element created at the $h$th hierarchical level is computed as
Fig. 1. Process of creating children elements in the HIFEM for a nonconforming three-node triangular element cut by two materials interfaces. Figures (b) and (c) illustrate the children elements associated with the first and second levels of hierarchy, respectively.

Fig. 2. Four different case scenarios for discretizing a nonconforming four-node tetrahedron element cut by a single materials interface into a combination of tetrahedron and wedge children elements in the IGFEM.

\[ s_j^{(b)} = \sqrt{\frac{\min(d_{ij})}{h_r}}, \]

where \( d_{ij} \) is the distance between node \( j \) and the \( l \)th edge of the child element and \( h_r \) is the size of the root element belonging to the background FE mesh.

3. 3D HIFEM algorithm

While a similar algorithm to that described in Section 2.2 can be implemented for evaluating the HIFEM enrichment functions for 3D problems, constructing appropriate children elements in such problems requires further considerations. In this manuscript, we focus on the HIFEM approximation for 3D problems discretized using nonconforming FE meshes composed of four-node tetrahedrons, although the proposed algorithm for creating children elements can be expanded to other types of elements as well. It must be noted that some of the computational geometry aspects discussed in this section can be also found useful in creating integration sub-elements in the 3D XFEM/GFEM.

As discussed in [23] and shown in Fig. 2, there are four distinct case scenarios for discretizing a four-node tetrahedron parent element into children elements in the IGFEM. As shown in that figure, in three cases wedge elements are used as children elements, which although simplifies the construction of sub-elements, may not be considered as an appropriate approach for creating children elements in the HIFEM. As noted in [28], implementing the recursive algorithm described in Section 2.2 for creating HIFEM children elements (and subsequently enrichment functions) requires constructing similar children elements at all levels of hierarchy. Thus, to take advantage of this characteristic, all the wedge elements used for evaluating IGEM enrichment functions must be replaced with tetrahedrons.

The process of discretizing and replacing a wedge subdomain with three tetrahedron children elements is schematically shown in Fig. 3(a). The first step is to select three cuts on quadrilateral sides of the wedge to create triangular surfaces of tetrahedrons. However, performing this task independently in each nonconforming element and without taking into account the position of the materials interface in adjacent elements would give rise to another problem, which is depicted in Fig. 3(b). This figure illustrates the tetrahedralization process for constructing children element in two adjacent parent elements. As shown there, the cuts on surface 5 shared between two adjacent elements are not compatible, which can lead to the construction of inappropriate enrichment functions and therefore the deterioration of the HIFEM accuracy. Note that HIFEM hierarchical enrichments are evaluated as a linear combination of Lagrangian shape functions in children elements, which must conform to one another over the surface of adjacent children elements.

To construct appropriate tetrahedral children elements, we perform a simple check before tetrahedralizing each wedge subdomain to determine whether a cut has already been made on surfaces of its adjacent elements. For computer implementation, this can be done by comparing the pair of equation numbers associated with diagonal nodes of a cut on each
resembling only results wedge inside adjacent if least from children subdomain element. the quadrilateral construction Fig. 4. Different this cuts Fig. 3. Tetrahedralization on the tetrahedron surface. (a) Tetrahedralization of a wedge subdomain into three tetrahedrons via selecting three appropriate cuts on its quadrilateral surfaces; (b) incompatible cuts on surface $S$ shared between two adjacent wedge subdomains, resulting in the construction of nonconforming tetrahedron children elements along this surface.

quadrilateral surface to detect existing cuts. We will then use these cuts on each wedge subdomain surface to construct the tetrahedron children elements and allow making new cuts only if no cut exists on the similar surface of the adjacent element. However, not having the freedom to pick the cut between two pairs of nodes on quadrilateral surfaces of a wedge subdomain can impede the construction of tetrahedron sub-elements. As shown in Fig. 4, there are eight different case scenarios for cutting these surfaces but only the six cases depicted in Fig. 4(a) allow the construction of the tetrahedral children elements. In contrary, it can be shown that for the cases illustrated in Fig. 4(b), where none of the cuts originate from the same node, it is impossible to create three tetrahedral subelements. Such cases can be avoided by ensuring that at least two cuts share the same node, which is always possible if having the freedom to make at least one new cut. However, if cuts on all three quadrilateral surfaces of the wedge subdomain are dictated by those already been made on surfaces of adjacent element, the formation of one of the cases shown in Fig. 4(b) might be inevitable.

Tetrahedralization of a wedge subdomain using either of the cuts shown in Fig. 4(b) requires creating an additional point inside the element. This so-called Steiner point will serve as an enriched node in HIFEM and will be used to discretize the wedge element into eight children subelements, as illustrated in Fig. 5. It must be noted that while adding a Steiner point results in an extra generalized dof and several additional children elements in the HIFEM, creating such a point would only be necessary in elements with inappropriate pre-existing cuts on surfaces of all their adjacent elements, i.e., cuts not resembling any of the cases shown in Fig. 4(a). Therefore, creating the Steiner point is only required in a small fraction of
finite elements of the nonconforming background mesh, which has a negligible effect on the overall computational cost of the HIFEM.

A naive algorithm for determining the location of the Steiner point is to use the centroid of the wedge subdomain, which ensures this point is located inside the wedge only if it is a convex pentahedron. However, this characteristic can be easily violated if the subdomain is created as a result of cutting the parent element with a curved materials interface. As schematically shown in Fig. 6, the centroid of such a concave polyhedron can be outside its boundaries and thus may not be used as an appropriate location for the Steiner point. It must be noted that the formation of concave polyhedrons with centroids outside their domains during the tetrahedralization process does not essentially require a significant variation of the curved interface slope in the parent element. This is because the HIFEM allows materials interfaces to intersect with edges of background elements at completely arbitrary locations and thus such cases can easily occur in polyhedrons with high aspect ratios, i.e., when the curved interface is considerably close to edges of the background mesh. In other words, refining the background FE mesh does not assure that the centroid of the concave polyhedron is confined within its boundaries and thus a more advanced algorithm must be implemented to determine the Steiner point location.

Finding the Steiner point coordinates such that it is not only located inside a concave polyhedral subdomain but also yields appropriate tetrahedral children elements gives rise to a geometric problem, which despite its complexity, can be solved via a computationally inexpensive procedure. The sketch of the proposed algorithm for determining the location of this point is depicted in Fig. 7. Note that, as shown in Fig. 7(a), discretizing a concave polyhedron into eight four-node tetrahedron children elements requires dividing its curved surface into two triangle-shaped surfaces (\(\Delta MNO\) and \(\Delta MNP\)). Therefore, the cut made on the curved surface for performing this task must be approximated as a straight line (cut MN) and thus the polyhedron subdomain is replaced with a concave hexahedron with straight edges. To evaluate the Steiner point location, we will first cut this hexahedron with a plane passing through points B and D corresponding to mid-points of edges MN and QR, respectively (Fig. 7(b)). This plane also intersects with two other edges of the hexahedron at points A and C, forming the concave quadrangle ABCD shown in Fig. 7(c). While any Steiner point on this quadrangle is guaranteed to be inside the concave hexahedron, it cannot necessarily be used as an appropriate location for tetrahedralizing the concave hexahedron. For example, as shown in Fig. 7(c), edges of the triangle created by using the Steiner point S1 (which is the cross-section of a tetrahedron element in 3D) are outside the quadrangle ABCD and therefore the resulting children element is not confined within the hexahedron boundaries.

To assure that all the eight tetrahedrons created using the Steiner point are located inside the concave hexahedron of Fig. 7(a), this point must be located in the quadrangle BFDE shown in Fig. 7(c). Points E and F shown in that figure are the intersection points between line segments BC–AD and AB–CD, respectively. Although there are myriads of choices for creating the Steiner point in this quadrangle, the midpoint of the line segment BD shown as point S2 in Fig. 7(c) is of particular interest. While guaranteed to be an appropriate Steiner point for tetrahedralizing the hexahedron, the point is not too close to any of the surfaces of this concave hexahedron, which avoids the construction of tetrahedron children elements with excessively large aspect ratios. However, the main reason for selecting the midpoint of BD as the Steiner point location
is the simple and computationally inexpensive procedure involved in evaluating the coordinates of this point. As noted earlier, the concave quadrangle ABCD of Fig. 7(c) is created by cutting the hexahedron with a plane passing through points B and D, i.e., midpoints of edges MN and QR, respectively (Fig. 7). Therefore, the coordinates of the Steiner point S2 can be computed as

$$\mathbf{X}_{S2} = 0.25(\mathbf{X}_M + \mathbf{X}_N + \mathbf{X}_Q + \mathbf{X}_R).$$

(10)

After the construction of children elements, the HIFEM enrichment functions can be evaluated using the same recursive algorithm described in Section 2.2, which requires repeating the tetrahedralization scheme proposed above at all levels of hierarchy.

4. Convergence study

In this section, we investigate the accuracy and convergence rate of the HIFEM for simulating the conductive heat transfer in a bi-material cube with the length of 1 mm, as shown in Fig. 8(a). The domain is insulated over its side surfaces, and has prescribed temperatures $u_b = 10^\circ C$ and $u_t = 0^\circ C$ over the bottom and top surfaces, respectively. Thermal conductivities of the two materials phases are $\kappa_1 = 50$ W/(m K) and $\kappa_2 = 5$ W/(m K). The curved interfaces depicted in Fig. 8(a) are segments of two ellipsoids with centroids located at $C_1 = (0, 0, 0)$ and $C_2 = (1, 1, 0)$ and similar radii of $R = (2, 1.5, 0.499)$ (the bottom left corner of the domain is located at the origin of the coordinate system). Due to the close proximity of materials interfaces, multiple elements of the backgrounds mesh are cut by both interfaces simultaneously (inset of Fig. 8(a)). The HIFEM approximation of the temperature field using a $31 \times 31 \times 31$ nonconforming structured FE mesh is depicted in Fig. 8(b).
Variations of the $L_2$- and $H^1$-norms of the error versus the element size $h$ for the HIFEM approximation of the thermal response of the problem shown in Fig. 8(a) and their comparison with standard FEM benchmark solutions using conforming FE meshes are illustrated in Fig. 9. The $L_2$ and $H^1$ norms of the error are computed as

$$E_{L_2(\Omega)} = \sqrt{\int_{\Omega} \| u - u^h \|^2 \, d\Omega},$$

$$E_{H^1(\Omega)} = \sqrt{\int_{\Omega} \left( \| u - u^h \|^2 + \| \nabla u - \nabla u^h \|^2 \right) \, d\Omega}. \tag{11}$$

A standard FEM simulation using a refined conforming FE mesh with $h_{\text{max}} = 0.005$ is adopted as the reference solution for evaluating the error. As shown in Fig. 9, the HIFEM yields the same level of accuracy and convergence rate as those of the standard FEM, while providing the advantage of eliminating the need to use conforming meshes.

5. Applications

In this section, we present the application of the 3D HIFEM for the mesh-independent simulation of thermal and mechanical behaviors of a particulate composite RVE and a ceramic matrix composite RVE, respectively.

5.1. Particulate composite

Dimensions, boundary conditions, and material properties of the particulate composite RVE studied in this example problem are illustrated in Fig. 10. Thermal conductivities of alumina inclusions and the glass matrix in this $1 \times 1 \times 0.5$ mm domain are $\kappa_A = 31$ W/(mK) and $\kappa_C = 5$ W/(mK), respectively. Boundary conditions consist of a prescribed temperature of $u_b = 0^\circ$C over the bottom surface, a constant heat flux of $q_t = 500$ W/m$^2$ applied over the top surface, and insulated lateral surfaces. Fig. 10(b) shows the distribution of percolating particles in the composite RVE. A small portion of the $60 \times 60 \times 30$ structured FE mesh used for discretizing the domain on section A–A is also depicted in the inset of Fig. 10(c).

The HIFEM approximation of the thermal response of the particulate composite RVE is depicted in Fig. 11(a). The temperature field along slice B–B and section C–C are also shown in Figs. 11(b) and 11(c), respectively. As shown in these figures, despite using a structured conforming FE mesh for constructing the discretized model, the HIFEM captures discontinuities of the temperature gradient along the interface between percolating alumina particles and the surrounding glass matrix.

5.2. Ceramic matrix composite

In this example, we implement the 3D HIFEM to simulate the deformation response and mechanical strains in a Ceramic Matrix Composite (CMC) RVE with the microstructure, materials properties, and boundary conditions shown in Fig. 12. CMCs have a high resistance against thermal loads and extreme environments, which makes them a suitable materials system for aerospace applications. However, the brittle mechanical behavior of ceramic is an undesirable property, which can lead to catastrophic failure events in such structures. To avoid this, a coating layer with lower mechanical strength (e.g., pyrocarbon) is deposited on ceramic fibers surfaces to provide a weak interface between fibers and the matrix. This coating layer controls the CMC failure mode by enforcing the fiber pull-out failure mechanism rather than the instantaneous crack propagation in brittle fiber/matrix phases.
Fig. 10. First application problem: (a) BCs and material properties; (b) distribution of ellipsoid particles inside the domain; (c) percolating particles and a portion of the background FE mesh and children elements along section A–A at $y = 0.5$ mm.

Fig. 11. First application problem: (a) HIFEM approximation of the temperature field using a $60 \times 60 \times 30$ structured FE mesh for discretizing the domain; (b) temperature field along the diagonal slice B–B; (c) temperature profile along section C–C at $z = 0.4$ mm, warped by a scale factor of 2.5.
The HIFEM approximation of the deformation response of the CMC RVE and the normal strain field in the transverse fibers direction are illustrated in Fig. 13. A 60 × 60 × 30 nonconforming structured FE mesh is employed to discretize the domain. As shown in Fig. 13, although the small thickness of the coating layer (hc = 50 μm) causes multiple elements of the background mesh to be cut by both the fiber-coating and coating-matrix interfaces, the HIFEM can properly simulate the discontinuous strain field in the CMC.

6. Conclusion

The formulation and implementation of the hierarchical interface-enriched finite element method (HIFEM) for simulating multiphase 3D problems was presented. This mesh-independent technique implements a hierarchical enrichment scheme applied via a recursive algorithm to capture weak discontinuities in nonconforming elements cut by arbitrary number and orientation of materials interfaces. These enrichment functions are created using a set of children elements constructed by cutting the nonconforming finite element with materials interfaces at different levels of hierarchy. A detailed discussion was provided on the proposed algorithm for creating HIFEM children elements in 3D meshes composed of four-node tetrahedral elements. Despite the complexity of computational geometry problems involved in tetrahedralizing such elements, we presented straightforward algorithms that can be implemented at a low computational cost for performing this task. A convergence study was provided to show that the 3D HIFEM yields a similar accuracy and convergence rate as those of the standard FEM but without the burden of creating conforming FE meshes. We also demonstrated some engineering applications of this method for simulating the conductive heat transfer in a particulate composite RVE and approximating the mechanical behavior of a ceramic matrix composite with pyrocarbon-coated ceramic fibers.

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