Data Collection and Analysis – Preparation

Before coming to class, do the following:


2. If you aren’t familiar with the terms mean, median, and mode (or need a review), review the presentation on the website.

2. Take the associated quiz on Carmen.
CHAPTER 6

Engineering Measurements and Estimations

Chapter Objectives

When you complete your study of this chapter, you will be able to:

- Determine the number of significant digits in a measurement
- Perform numerical calculations with measured quantities and express the answer with the appropriate number of significant digits
- Define accuracy and precision in measurements
- Define systematic and random errors and explain how they occur in measurements
- Solve problems involving estimations of the required data and assumptions to enable a solution
- Develop and present problem solutions, involving finding or estimating the necessary data, that enable others to understand your method of solution and to determine the validity of the numerical work

6.1 Introduction

The nineteenth-century physicist Lord Kelvin stated that knowledge and understanding are not of high quality unless the information can be expressed in numbers. Numbers are the operating medium for most engineering functions. In order to perform analysis and design, engineers must be able to measure physical quantities and express these measurements in numerical form. Furthermore, engineers must have confidence that the measurements and subsequent calculations and decisions made based on the measurements are reasonable.

In this chapter, we will describe how to properly use measurements (numbers) in engineering calculations. In your specific engineering discipline you will gain experience in selecting the correct measuring device for a particular situation.

6.2 Measurements: Accuracy and Precision

In measurements, “accuracy” and “precision” have different meanings and cannot be used interchangeably. Accuracy is a measure of the nearness of a value to
the correct or true value. Precision refers to the repeatability of a measurement, that is, how close successive measurements are to each other. Figure 6.1 illustrates accuracy and precision of the results of four dart throwers. Thrower (a) is both inaccurate and imprecise because the results are away from the bull’s-eye (accuracy) and widely scattered (precision). Thrower (b) is accurate because the throws are evenly distributed about the desired result but imprecise because of the wide scatter. Thrower (c) is precise with the tight cluster of throws but inaccurate because the results are away from the desired bull’s-eye. Finally, thrower (d) demonstrates accuracy and precision with tight cluster of throws around the center of the target. Throwers (a), (b), and (c) can improve their performance by analyzing the causes for the errors. Body position, arm motion, and release point could cause deviation from the desired result.

As engineers perform computations in analysis and design, the accuracy and precision of data gathered for the computations must be ascertained. For a quantity that is measured by a physical instrument, the exact numerical value of the quantity is likely to remain unknown. Therefore the measurement must be recorded with the known limitations in accuracy and precision taken into account. To do this engineers apply accepted practices and rules and carefully note the conditions under which the data was obtained. The practices and rules for performing numerical computations are discussed in Section 6.3. A brief discussion of the identification of errors in measurements is found in Section 6.4.
6.3 Measurements: Significant Digits

Numbers used for calculation purposes in engineering design and analysis may be integers (exact) or real (exact or approximate). For example, six one-dozen cartons of eggs include a countable number of eggs, exactly 72. There are 2.54 centimeters in one inch, an example of an exact real number. Thus if the conversion of inches to centimeters is required in a calculation, 2.54 is not a contributing factor to the precision of the result of the calculation. The ratio of the circumference of a circle to its diameter, \( \pi \), is an approximate real number that may be written as 3.14, 3.142, 3.14159, \ldots, depending on the precision required in a numerical calculation.

Any physical measurement that is not a countable number will be approximate. Errors are likely to be present regardless of the precautions used when making the measurement. Let us look at measuring the length of the metal bar in Figure 6.2 with a scale graduated in tenths of inches. At first glance it is obvious that the bar is between 2 and 3 inches in length. We could write down that the bar is \( 2.5 \pm 0.5 \) inches long. Upon closer inspection we note the bar is between 2.6 and 2.7 inches in length, or \( 2.65 \pm 0.05 \) inches. What value would we use in a computation? 2.64? 2.65? 2.66? We might select 2.64 as the "best" measurement, realizing that the third digit in our answer is doubtful and therefore our measurement must be considered approximate.

It is clear that a method of expressing results and measurements is needed that will convey how "good" these numbers are. The use of significant digits gives us this capability without resorting to the more rigorous approach of computing an estimated percentage error to be specified with each numerical result or measurement. Before we introduce significant digits, it is necessary to discuss the presentation of numerical values in formats that leave no doubts in interpretation.

The following are accepted conventions for the presentation of numbers in engineering work:

1. For numbers less than one, a zero is written in front of the decimal point to omit any possible errors due to copy processes or careless reading. Therefore, we write 0.345 and not .345.
2. A space, not a comma, is used to divide numbers of three orders of magnitude or more. We write 4,567.8 instead of 4,567.8 and 0.678 91 instead of 0.678,91.

**Figure 6.2**

![Measurement Scale](image)

Making an approximate measurement.
3. For very large or very small numbers we use scientific notation to reduce the unwieldy nature of these numbers. For example, supercomputer calculating rates are compared by using the Linpack benchmark performance criteria. One of the criteria used by Linpack is the solving of a very large set of simultaneous linear equations. In the 2009 rankings of supercomputers, an IBM/DOE supercomputer named BlueGene/L, installed at the Lawrence Livermore National Laboratory in California, was benchmarked at 478 200 000 000 000 flops/s, seventh best sustainable performance in the world. The unit flops/s stands for floating-point operations per second, which is computer terminology for calculations using real numbers. In scientific notation this number would be $478.2 \times 10^{12}$, an obviously more compact representation. We will see that scientific notation is of great assistance in the determination and representation of significant digits.

Another convenient method of representing measurements is with prefix names that denote multipliers by factors of 10. Table 6.1 illustrates decimal multipliers and their corresponding prefixes and symbols. Thus the BlueGene/L computer performance could also be represented as 4782 teraflops/s, or 478.2 Tiflops/s. The highest computational performance in the world in 2009 was established by the Jaguar, a Cray XT5-HE Opteron system, measured at 1 759 Tflops/s or 1.759 Pflops/s.

The use of prefixes enables us to express any measurement as a number between 0.1 and 1 000 with a corresponding prefix applied to the unit. For example, it is clearer to the reader if the distance between two cities is expressed as 35 kilometers (35 km) rather than 35 000 meters (35 000 m). More on prefixes may be found in the Chapter 7 discussion of the International System of Units (SI).

A significant digit, or significant figure, is defined as any digit used in writing a number, except those zeros that are used only for location of the decimal point.

<table>
<thead>
<tr>
<th>Multiplier</th>
<th>Prefix name</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{18}$</td>
<td>exa</td>
<td>E</td>
</tr>
<tr>
<td>$10^{15}$</td>
<td>peta</td>
<td>P</td>
</tr>
<tr>
<td>$10^{12}$</td>
<td>tera</td>
<td>T</td>
</tr>
<tr>
<td>$10^9$</td>
<td>giga</td>
<td>G</td>
</tr>
<tr>
<td>$10^6$</td>
<td>*mega</td>
<td>M</td>
</tr>
<tr>
<td>$10^3$</td>
<td>*kilo</td>
<td>k</td>
</tr>
<tr>
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<td>h</td>
</tr>
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<td>da</td>
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<td>$10^{-1}$</td>
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<tr>
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<td>m</td>
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<td>µ</td>
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<td>femto</td>
<td>f</td>
</tr>
<tr>
<td>$10^{-18}$</td>
<td>atto</td>
<td>a</td>
</tr>
</tbody>
</table>

*Most often used.
### Figure 6.3

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Number of Significant Figures</th>
</tr>
</thead>
<tbody>
<tr>
<td>4784</td>
<td>4</td>
</tr>
<tr>
<td>36</td>
<td>2</td>
</tr>
<tr>
<td>60</td>
<td>1 or 2</td>
</tr>
<tr>
<td>600</td>
<td>1, 2, or 3</td>
</tr>
<tr>
<td>$6.00 \times 10^2$</td>
<td>3</td>
</tr>
<tr>
<td>31.72</td>
<td>4</td>
</tr>
<tr>
<td>30.02</td>
<td>4</td>
</tr>
<tr>
<td>46.0</td>
<td>3</td>
</tr>
<tr>
<td>0.02</td>
<td>1</td>
</tr>
<tr>
<td>0.020</td>
<td>2</td>
</tr>
<tr>
<td>600.00</td>
<td>5</td>
</tr>
</tbody>
</table>

Point or those zeros that do not have any nonzero digit on their left. When you read the number 0.001 5, only the digits 1 and 5 are significant, since the three zeros have no nonzero digit to their left. We would say this number has two significant figures. If the number is written 0.001 50, it contains three significant figures; the rightmost zero is significant.

Numbers 10 or larger that are not written in scientific notation and that are not counts (exact values) can cause difficulties in interpretation when zeros are present. For example, 2000 could contain one, two, three, or four significant digits; it is not clear which. If you write the number in scientific notation as $2.000 \times 10^3$, then clearly four significant digits are intended. If you want to show only two significant digits, you would write $2.0 \times 10^3$. It is our recommendation that if uncertainty results from using standard decimal notation, you switch to scientific notation so your reader can clearly understand your intent. Figure 6.3 shows the number of significant figures for several quantities.

You may find yourself as the user of measurements where the writer was not careful to properly show significant figures. Assuming that the number is not a count or a known exact value, you should establish a reasonable number of significant figures based on the context of the measurement and on your experience. Once you have decided on a reasonable number of significant digits in each measured value, you can then use that number in any calculations that are required making sure you clearly explain your actions.

When an instrument, such as an engineer's scale, analog thermometer, or fuel gauge, is read, the last digit will normally be an estimate. That is, the instrument is read by estimating between the smallest graduations on the scale to get the final digit. In Figure 6.4a, the reading is between 1.2 and 1.3, or 1.25 ± 0.05. For calculation purposes we might select 1.27 as a best value, with the 7 being a doubtful digit of the three significant figures. It is standard practice to count one doubtful digit as significant, thus the 1.27 reading has three significant figures. Similarly, the thermometer reading in Figure 6.4b is noted as 52.5° ± 0.5, and we estimate a best value of 52.8° with the 8 being doubtful.

In Figure 6.4c, the graduations create a more difficult task for reading a fuel level. Each graduation is one-sixth of a full tank. The reading is between
Reading graduations on instruments will include a doubtful or estimated value.

1/6 and 2/6 full, or 3/12 ± 1/12. How many significant figures are there? If we convert the reading to 0.25 ± 0.0833, a "best" estimate might be 0.30. In any case you cannot justify more than one significant figure and the answer would be expressed as 0.3. The difficulty in this example is not the significant figures but the scale of the fuel gauge. It is meant to convey a general impression of the fuel level and not a numerically significant value. Therefore, the selection of the instrument is an important factor in physical measurements.

Calculators and computers maintain countable numbers (integers) in exact form up to the capacity of the machine. Real numbers are kept at the precision level of the particular device. This is true no matter how many significant digits an input value or calculated value should have. Therefore, you will need to exercise care in reporting values from a calculator display or from a computer output (a spreadsheet for example). Calculators and most high-level computer languages allow you to control the number of digits that are to be displayed or printed. If a computer output is to be a part of your final solution presentation, you will need to carefully control the output form. If the output is only an intermediate step, you can round the results to a reasonable number of significant figures in your presentation.

As you perform arithmetic operations, it is important that you not lose the significance of your measurements or, conversely, imply precision that does not exist. Rules for determining the number of significant figures that should be reported following computations have been developed by engineering associations. The following rules customarily apply.

1. **Rounding.** General Rule: Round a value to the proper number of significant figures, increase the last digit retained by 1 if the first figure dropped is 5 or greater. This is the rule normally built into a calculator display control or a control language.

   **Examples**
   
a.  23.650 rounds to 23.7 for three significant figures.
   b.  0.0143 rounds to 0.014 for two significant figures.
   c.  827.48 rounds to 827.5 or 827 for four and three significant digits, respectively.

   *(Note: You must decide the number of significant figures before you round. For example, rounding 827.48 to three significant figures yields*
827. However, if you first round to four figures, obtaining 827.5, and then round that number to three figures, the result would be 828—which is not correct.

2. **Multiplication and Division.** General Rule: The product or quotient should contain the same number of significant digits as are contained in the number with the fewest significant digits.

**Examples**

a. \((2.43)(17.675) = 42.95025\)
   If both the multiplier and multiplicand are exact, the answer should be reported as 42.95025. If one or both of the numbers are not exact, as is normally the case, the rule is applied using the inexact number with the fewest significant figures. In this example, assuming both numbers are inexact, 2.43 has three significant figures and 17.675 has five. Applying the rule, the answer should contain three significant figures and be reported as 43.0 or \(4.30 \times 10^1\).

b. \((2.479 \text{ h})(60 \text{ min/h}) = 148.74 \text{ min}\)
   In this case, the conversion factor is exact (a definition) and could be thought of as having an infinite number of significant figures (60.00000...). Thus, 2.479, which has four significant figures, controls the precision, and the answer is 148.7 min, or \(1.487 \times 10^2 \text{ min}\).

c. \((4.00 \times 10^3 \text{ kg})(2.2046 \text{ lbm/kg}) = 881.84 \text{ lbm}\)
   Here, the conversion factor is not an exact number, but you should not let the conversion factor dictate the precision of the answer if it can be avoided. You should attempt to maintain the precision of the value being converted \((4.00 \times 10^3)\); you cannot improve its precision. If you need to maintain precision greater than what is available in the conversion factor, we recommend that you locate or calculate the conversion factor to one or two more significant figures than the value you are converting. For example, the conversion factors given in the appendix of this text contain five significant figures. If you require more for a calculation, you would need to locate a more precise conversion factor from another source or derive the conversion factor and calculate it to the desired number of significant figures. For example (c), three significant figures should be reported, yielding 882 lbm.

d. \(589.62/1.246 = 473.21027\)
   The answer, to four significant figures, is 473.2.

3. **Addition and Subtraction.** General Rule: The answer should show significant digits only as far to the right as is seen in the least precise number in the calculation. Remember that the last number recorded is doubtful, that is, an estimate.

**Example**

a. \[
\begin{align*}
& 1725.463 \\
& 189.2 \\
& 16.73 \\
\end{align*}
\]
\[
\begin{align*}
\hline
& 1931.393 \\
\end{align*}
\]
The least precise number in this group is 189.2 because the (2) is an estimate, so, according to the rule, the answer should be reported as 1 931.4. Using alternative reasoning, suppose these numbers are instrument readings, which means the last reported digit in each is a doubtful digit. A column addition that contains a doubtful digit will result in a doubtful digit in the sum. As a result, all three digits to the right of the decimal in the answer are doubtful. We keep just one doubtful digit in the answer; thus the answer is 1 931.4 after rounding.

\[ 897.0 \]
\[ - 0.092 \ 2 \]
\[ 896.907 \ 8 \]

Application of the rule results in an answer of 896.9.

4. **Combined Operations.** General rule: If products or quotients are to be added or subtracted, perform the multiplication or division first, establish the correct number of significant figures in the intermediate answer, then perform the addition or subtraction, and round to proper significant figures. Note, however, that in calculator or computer applications it is not practical to perform intermediate rounding. It is normal practice to perform the entire calculation and then report a reasonable number of significant figures.

If results from additions or subtractions are to be multiplied or divided, an intermediate determination of significant figures can be made when the calculations are performed manually. For calculator or computer answers, use the suggestion already mentioned.

Subtractions that occur in the denominator of a quotient can be a particular problem when the numbers to be subtracted are very nearly the same. For example, \( \frac{39.7}{(772.3 - 772.26)} \) gives 992.5 if intermediate rounding is not done. If, however, the subtraction in the denominator is reported with one digit to the right of the decimal, the denominator becomes zero and the result becomes undefined. Common sense application of the rules is necessary to avoid problems.

6.4 **Errors**

It is important to realize that all measurements of physical quantities have a certain amount of uncertainty (error) associated with them. This error must be made as small as possible. Therefore we must determine, as best we can, what errors are present and account for them in the measurement. Consider the thermometer in Figure 6.4b. After we make the reading of 52.8°, can we say for sure that this is the true temperature with the 8 being a doubtful digit? The answer is no. For example, what if the thermometer has not been calibrated properly and reads high by a degree or two? Maybe the thermometer has been smudged with oil, dirt, or grease and a mistake is made in reading the calibrations. Perhaps the sensor connecting the environment being measured to the thermometer is not connected properly. These and other known error possibilities must be accounted for to obtain an acceptable accuracy in the reading.
However, even if you carefully account for possible errors, the measurement will still have some error present. Depending upon the level of accuracy and preciseness required in the measurement, further effort may be needed in error determination. To clarify the presence of error, measurements can be expressed in two parts, (1) a number representing a mean value of the physical quantity measured, and (2) an amount of doubt (error) in this mean value. The amount of doubt (error) provides the accuracy of the measurement. For example, our thermometer reading in Figure 6.4b could be expressed as 52.5 ± 0.5, which brackets the upper and lower limits of the measurement based on the calibration of the instrument. To perform computations with measurements we carefully estimate a “best” value of 52.8 with the 8 being in doubt. You should note that error (deviation from the true temperature) is still present and that the true temperature is not known exactly.

A common application of measurement error is in pressure gauges. Suppose a gauge on an air tank is labeled ±2% at 150 lb/in². The range of the gauge error would be (0.02)(150) = 3.0 lb/in². Therefore a reading of 190 would indicate that the true pressure falls between 187 and 193 lb/in².

Errors can be classified into two broad categories for analysis: systematic and random.

### 6.4.1 Systematic Errors

A systematic error tends to shift a measurement consistently in the same direction from the true value. Examples include errors in the calibration of the measuring instrument, failure to account for some external effect on the instrument and improper use of the instrument. Large systematic errors must be corrected in any situation. Consider a 25 m steel tape that is compared with the standard in the U.S. Bureau of Standards in Washington, D.C. If the tape is not exactly 25,000 m long then each time the tape is used, the known error can be incorporated in the measurement. Similarly large temperature changes in a steel tape can be corrected by a known coefficient of thermal expansion. However, if all known systematic errors are accounted for, there always remain small errors. For example, no instrument can be calibrated perfectly.

### 6.4.2 Random Errors

Random errors are those that fluctuate from one measurement to another for the same instrument. One cause is the sensitivity of the instrument. A small change in the quantity measured may not be picked up by the instrument. Such errors are usually distributed equally around the true value. Another random error may occur when an instrument is read by more than one person. Consider a water barometer that measures atmospheric pressure. Close inspection of the water level shows a meniscus which can easily result in different readings from several observers. These readings will very likely divide above and below a mean, or true, value. Statistical analysis is used to provide some insight into random errors. Chapters 10 and 11 provide an introduction to the statistical concepts of central tendency which are a part of the analysis of random errors.
6.5 Estimations

Engineers strive for a high level of precision in their work. However, it is also important to be aware of an acceptable precision and the time and cost of attaining it. There are many instances where an engineer is expected to estimate the result to a problem with reasonable accuracy but under tight time and cost constraints. To do this engineers rely on their basic understanding of the problem under discussion coupled with their previous experience. This knowledge and experience is what distinguishes an “estimation” from a “guess.” If greater accuracy is needed, the initial estimation can be refined when time, funds, and the necessary additional data for refining the result are available.

Initial estimates may be in error by perhaps 10 to 20% or even more. The accuracy of these estimates depends strongly on what reference materials we have available, how much time is allotted for the estimate, and, of course, how experienced we are with similar problems. The first example we present will attempt to illustrate what an engineer might be called upon to do in a few minutes with little in the way of references.

Example Problem 6.1  An aerospace engineer is asked to sit in on a meeting of executives of an airline considering the purchase of the new Boeing 787 Dreamliner. The engineer is asked if she could give the group a quick estimate of the average cruise fuel consumption of the 787 on a per-mile basis. The executives can use the result to compare the 787 with competitive aircraft as they proceed toward a purchase decision.

Discussion  The engineer has reviewed preliminary specifications published by Boeing to prospective buyers of the 787. The base model of the aircraft (787-8) has an estimated range of 7 650 to 8 200 nautical miles (nm), or 8 803 to 9 436 miles, and a cruise Mach number of \( M = 0.85 \). Mach number is the ratio of the speed of the aircraft to the speed of sound at the flight altitude. The fuel capacity is about 33 000 gallons or about 220 000 lb. Assuming that 10% of the fuel is used during taxi, takeoff, climb to cruise altitude, descent, and final taxi and a 10% reserve is required upon arrival, and using the high end of the mileage range, the engineer quickly estimates cruise fuel consumption per mile as 26 400\( /9 \) 436 = 2.8 gal/mi or 176 000\( /9 \) 436 = 18.7 lb/mi. From this quick estimate, the executives can estimate a fuel cost per mile at current fuel prices and the overall fuel cost for any flight leg. This problem does not require specialized knowledge in any particular engineering discipline, but does require an ability to understand the problem and put together the available data to obtain a solution. Experience in the aerospace discipline speeds the understanding of the problem and enables the quick estimate to be done. Some additional data and estimation problems involving the 787 are presented in the chapter problems.

Example Problem 6.2 is an illustration of a problem you might be assigned. You have the necessary experience to perform the estimation without special knowledge. Not counting the final written presentation, you should be able to do a similar problem in one-half to 1 hour.
The Boeing 787 Dreamliner

The latest commercial aircraft being designed, developed, and manufactured by The Boeing Company is the 787 Dreamliner. This aircraft is in response to preferences expressed by airlines around the world for a super-efficient airplane. The 787 is the ninth airplane in the Boeing 7x7 commercial fleet, which began in the 1950s with the 707 and continued with the 727, 737, 747, 757, 767, 777, and now the 787, due to go into service in late 2010.

The 787 family will initially consist of two models. The 787-8 will carry 210–250 passengers on routes of 7500 to 8000 nautical miles (nm), or 14 200 to 15 200 kilometers (km). The 787-8 was test-flown for the first time on December 15, 2009. Delivery to customers is expected to begin in late 2010. The second family member is the 787-9. This Dreamliner will carry 250–290 passengers on routes of 8 000 to 8 500 nm (14 800 to 15 750 km). Each of the family members is specifically optimized for its design range, passenger capacity, and cargo capacity.

Advancing technologies, design procedures, and manufacturing improvements have led to an aircraft of unprecedented efficiency. The primary structure of the aircraft will be 50% composites and 20% aluminum, compared with 12% composites and 50% aluminum on the 777, which went into service in 1995. The great increase in composites will eliminate 1 500 aluminum sheets and 40 000 to 50 000 fasteners per fuselage section, thus saving assembly time, material handling, and weight. The empty weight of the 787 will run 40 000 to 50 000 lb less than other aircraft in its class, giving airlines added cargo revenue. Maximum takeoff weight consists of empty aircraft plus fuel weight plus cargo (passengers and freight). If the empty weight can be reduced, cargo can increase correspondingly for the same takeoff weight, generating increased revenue.

The Rolls-Royce Trent 1000 and the General Electric GEnx engines, each rated at 64 000 lb of thrust have been selected for installation on the 787-8. Advances in engine technology will create an 8% increase in efficiency and the engines will use up to 20% less fuel for comparable routes than any other aircraft of similar size. Engine emissions are therefore reduced by an equivalent amount. The engines will propel the aircraft at speeds equivalent to current large jets, Mach 0.85.

Another major area of improvement is the replacement of pneumatic systems with electrical systems to run auxiliary devices. In previous aircraft, bleed air was taken from the engine compressor to, among other uses, power the pneumatic systems for cabin pressure. Now the engines will drive electric generators that will in turn provide the power for the auxiliary devices. This results in a 35% drop in the power extracted from the engines.

Passengers will experience improved conditions for flights. The Dreamliner’s windows are 65% larger than those in competitive aircraft. Climate control in the cabin will include higher humidity and improved air handling. Seats are ergonomically designed to provide improved comfort and convenience.

The 787 Dreamliner is truly an international effort. The Boeing Commercial Airplanes in the Seattle, Washington, area will handle 787 development integration, final assembly, and program leadership. Major components of the aircraft will be manufactured in 26 foreign countries, including Japan (wing box and main landing gear well), France (landing gear structure and electric brakes), the United Kingdom (landing gear actuation and control system and Rolls-Royce engines), Korea (raked wing tips), Sweden (cargo and access doors), and Germany (main cabin lighting and metal tubing and ducting). Other contributing companies in the United States include Boeing subsidiaries in Washington and Kansas, Rockwell Collins in Iowa, Honeywell in Arizona, Goodrich in North Carolina, General Electric in Ohio (engine), Moog Inc. in New York, and Hamilton Sundstrand in Connecticut.
Example Problem 6.2  Suppose your instructor assigns the following problem: Estimate the height of two different flagpoles on your campus. This will be done on a cloudy day so no shadow is present. The poles are in the ground (not on top of a building) and the bases of the poles are accessible. One of the poles is on level ground and you have available a carpenter’s level, straight edge, protractor, and masking tape. The other pole is on ground that slopes away from the base and you have available a carpenter’s level, straight edge, protractor, and a 12-ft. tape. See Figure 6.5 for the response of one student (whom we will call Dave).

Discussion  To estimate the height of the flagpole on level ground, Dave recognizes that he does not have a normal distance measuring device but that he must know some distance in order to use trigonometry to solve the problem. He knows that he is 6'1" tall, and he can mark that height on the pole with masking tape, which he can see from a location several feet from the pole. He can use the level, protractor, and straight edge to estimate angles from the horizontal. The distance away from the pole for measuring the angles is arbitrary, but he chooses a distance that will provide angles that are neither too large nor too small, both of which would be difficult to estimate. Then from this point on the ground he estimates the angles to his 6'1" masking tape mark and to the top of the pole. Note that Dave has kept all of the significant figures through the calculations and rounded only at the end, reasoning that he does not want intermediate rounding to affect his answer. Based on the method used for estimation, he believes his answer is not closer than the nearest foot so he rounds to that value.

When estimating the height of the pole on sloping ground, Dave had a tape available so it was not necessary to mark his height on the pole. Again, Dave kept all significant figures through the calculation procedure and rounded only the final result.

Example Problem 6.3  A homeowner has asked you to estimate the number of gallons of paint required to prime and finish-coat her new garage. You are told that paint is applied about 0.004 in. thick on smooth surfaces. The siding is to be gray and the roof overhang and trim are to be white. See Figure 6.6 for one approach done by Laura.

Discussion  From experience, Laura knew that one coat of primer would be needed and that two coats of finish paint would be required for a lasting outcome. She also noted that the garage doors were painted by the manufacturer before installation and therefore would not need further paint. She decided to neglect the effect of windows in the garage because of their small size. She observed that the siding was a rough vertical wood type and that the soffit (underside of the roof overhang) was smooth plywood. Since she had limited experience with rough siding, she contacted a local paint retailer and learned that rough siding would take approximately three times as much primer as smooth siding and that because of the siding roughness the finish paint would cover only about 3/4 of the normal area. She carefully documented this fact in her presentation.
### Figure 6.5a

<table>
<thead>
<tr>
<th>9 - 24 - X</th>
<th>ENGR 160</th>
</tr>
</thead>
<tbody>
<tr>
<td>FLAGPOLE PROBLEM</td>
<td>DAVE DOE</td>
</tr>
</tbody>
</table>

**Problem 5.2**

Estimate the height of 2 flagpoles on your campus. Assume the sun is not shining and that the bases of the poles are at ground level (not on top of a building) and that the bases are accessible.

A. Do the estimate for pole 1 where you have available a carpenter's level, straight edge, protractor and masking tape. Pole 1 sits on level ground.

B. For pole 2, you have a carpenter's level, straight edge, protractor and a 12' tape. The ground around pole 2 slopes away from the base.

**Part A**

**Assumptions**
1. Ground approximately level around flagpole base

**Procedure**
- Place a piece of masking tape at my height on the flagpole.
- Choose position about flagpole height away from the base and measure elevation angles to top of pole ($\beta$) and to tape ($\alpha$). See Fig. 1.

**Data**
1. My height known to be 6' 1''
2. Level, straight edge, and protractor can serve as a system for measuring elevation angles (Fig. 2)
3. $\alpha = 7.5^\circ$, $\beta = 48^\circ$

**Solution**

\[
\begin{align*}
\tan \alpha &= \frac{6'1''}{d} = \tan 7.5^\circ \\
\tan \beta &= \frac{h}{d} = \tan 48^\circ \\
\text{Approximate} \ h &= 51.319 \text{ ft}
\end{align*}
\]

Student presentation for Example Problem 6.2.
Figure 6.5b

**ASSUMPTIONS**

1. Ground has constant slope from base to measuring point.

**PROCEDURES**

- Choose point about flagpole height away from base and measure elevation angles to top ($\gamma$) and to base ($\Delta$).
- Measure distance ($d$) from chosen point to base. Fig. 3.

**DATA**

1. $\Delta = 3.5^\circ$
2. $\gamma = 44^\circ$
3. $d = 93.4''$

**SOLUTION**

\[
\sin \Delta = \frac{b}{d}
\]

\[
b = d \sin \Delta = (93.4'') \sin 3.5^\circ = 5.6979'
\]

\[
\tan \Delta = \frac{b}{d_h}
\]

\[
d_h = \frac{5.6979}{\tan 3.5^\circ} = 93.1598'
\]

\[
\tan \gamma = \frac{b + h}{d_h}
\]

\[
h = d_h \tan \gamma - b = (93.1598) \tan 44^\circ - 5.6979 = 84.265'
\]

**APPROXIMATE** $h = 84'$

**DISCUSSION/CONCLUSIONS**

1. The accuracy of the estimated heights could be verified if at least two other students made independent measurements with the same instruments.
2. Total time required: 70 minutes to set up the procedure, take measurements and do the calculations; 35 minutes for write-up.
PROBLEM 5.3

A HOMEOWNER HAS ASKED FOR AN ESTIMATE OF THE NUMBER OF GALLONS OF PAINT REQUIRED TO PRIME AND FINISH COAT HER NEW GARAGE. PAINT SHOULD BE APPLIED ABOUT 0.004 IN. THICK ON SMOOTH SURFACES. THE SIDING IS TO BE GRAY AND THE TRIM WHITE.

ASSUMPTIONS

1. 1 COAT OF PRIMER, 2 FINISH COATS
2. GARAGE DOORS ARE NOT PAINTED.
3. NEGLECT AREA OF SMALL WINDOWS IN GARAGE.

PROCEDURE

MEASURE GARAGE SURFACES TO OBTAIN TOTAL AREA TO BE PAINTED. OBSERVE "SMOOTHNESS" OF SIDING TO ESTIMATE PAINT COVERAGE, COMPUTE AMOUNT OF EACH TYPE OF PAINT.

COLLECTED DATA

1. SINCE 1 GAL = 231 IN³, PAINT THICKNESS OF 0.004 IN. RESULTS IN 1 GAL Covering = 400 FT² OF SMOOTH SURFACE.
2. OVERHANGS ARE 18 IN.
3. SIDING IS OBSERVED TO BE ROUGH WOOD/VERTICAL SOFFIT IS SMOOTH PLYWOOD.
4. LOCAL PAINT STORE REPRESENTATIVE SUGGESTED THAT PRIMER, ON ROUGH WOOD SIDING COVERS ONLY 1/3 NORMAL AREA AND THAT TOP COAT COVERS ABOUT 3/4 NORMAL AREA.

SOLUTION

NORTH SIDE AREA = (9)(50) - (7)(8) - (7)(16) - (7)(3) = 261 FT²
SOUTH SIDE AREA = (9)(50) = 450 FT²
EAST/WEST END AREA = 9(24) + 1/2(24)(6) = 276 FT² / END
OVERHANG AREA = (53)(15)(2) + 2(2)(15)[(13.5)² + (5/12)(13.5)]²/²
               = 159 + 27.75 = 247 FT²
<table>
<thead>
<tr>
<th>3-16-XX</th>
<th>ENGR 161</th>
<th>PROBLEM 5.3</th>
<th>LAURA LYNN</th>
</tr>
</thead>
</table>

TOTAL AREA OF SIDING = 261 + 450 + 2(276) = 1263 FT²

TOTAL OVERHANG AREA = 247 FT²

PRIMER NEEDED FOR SIDING = \( \frac{1263}{400} \times 3 \approx 9.47 \text{ GAL} \)

PRIMER NEEDED FOR OVERHANG = \( \frac{247}{400} \approx 0.62 \text{ GAL} \)

TOTAL PRIMER NEEDED = 9.47 + 0.62 = 10.1 GAL

GRAY FINISH COAT FOR SIDING = (2) \( \frac{1263}{400} \times \frac{1}{2} \) = 8.42 GAL

WHITE FINISH COAT FOR OVERHANG/TRIM = (2) \( \frac{247}{400} \) = 1.24 GAL

RECOMMENDED PURCHASE:

- PRIMER: 10 GAL
- GRAY TOP COAT: 9 GAL
- WHITE TOP COAT: 2 GAL

DISCUSSION/CONCLUSIONS

1. HOMEOWNER SHOULD BE INFORMED THAT HER PAINTING EXPERIENCE MAY AFFECT THE AMOUNT OF PAINT REQUIRED. MAINTAINING A CONSISTENT THICKNESS IS DIFFICULT.

2. A FRIEND HELPED ME OBTAIN THE MEASUREMENTS USING A 20 FT STEEL TAPE.

3. TOTAL TIME REQUIRED; 25 MINUTES FOR MEASUREMENTS, A STEP-LADDER WAS NEEDED; 40 MINUTES FOR CALCULATIONS AND WRITEUP.
Laura took all necessary measurements and computed the areas that must be painted. She determined that the paint film thickness of 0.004 in. corresponds to approximately 400 ft²/gal coverage. Like Dave in the previous example, Laura retained extra significant figures until finally rounding at the end of the estimate. In this case, except for the primer, she correctly rounded up rather than to the nearest gallon, as the paint would be purchased in whole gallons.

**Example Problem 6.4** Estimate the cost of the concrete that is in the roadbed of Interstate 17 running from Flagstaff, Arizona, to Interstate 10 near the Phoenix airport. Consider only the actual roadbed and not stopping lanes or interchanges. Keep track of the time to develop a solution and perform the write-up.

**Discussion** Figure 6.7 is a write-up on a word processor of the solution performed. The assumptions are listed and simplify the data collection process considerably. Two telephone calls were made to obtain information on interstate highway construction and the cost of concrete. Thus the resulting estimate is based on current design practice and costs. A similar estimation problem relaxing some of the assumptions is provided in Problem 6.29.

**Problems**

6.1 How many significant digits are contained in each of the following quantities?

(a) 0.72470 
(b) 7.2470 
(c) 0.031 
(d) 24,000 
(e) 0.10 \times 10^4 
(f) 0.3200 
(g) 200.07 
(h) 1.3200 \times 10^{-3} 
(i) 2,420,000.0 
(j) 3.0267 \times 10^2

6.2 How many significant digits are contained in each of the following quantities?

(a) 2.0345 
(b) 0.10030 
(c) 0.023 
(d) 2.2046 kg/lb 
(e) 300.030 
(f) 2.54 cm/in. 
(g) 1.002 
(h) 1.000 \times 10^6 
(i) 60 s/min 
(j) 4.030 \times 10^{12}

6.3 Perform the computations below and report with the answer rounded to the proper number of significant digits.

(a) 56.3 \times 372.5 
(b) 37.35 - 1.4300 
(c) (6.231827)(4.23 \times 10^7) 
(d) 2.5 / 0.50 
(e) 31.05 / 2.0 
(f) 1.456 \times 10^9(1.03 \times 10^{-5}) 
(g) 1.8457 \times 0.70025 
(h) 17.54678 / 0.02435 
(i) 4300 \div 784 
(j) (450.3 + 372 - 112.1) / 86.00

6.4 Using the conversion factors given in each problem, perform the calculations below using exact conversions or with enough significant digits that it does not affect the accuracy of the answer.

(a) 76200 feet to miles 
(b) 3.4 radians to degrees 
(c) 650 kg to pound mass 
(d) 7.358 inches to centimeters 
(e) 7.062 cubic meters to cubic feet 
(f) 12.5 days to seconds 

1 mi = 5280 ft 
1 radian = 57.296 degrees 
1 kg = 2.2046 lbm 
1 in. = 2.54 cm 
1 m = 3.2808 ft 
1 day = 86400 s
Estimate the cost of concrete in the roadbed of the entire length of Interstate Highway 17 (I-17), from its beginning at Flagstaff, Arizona to its terminus at Interstate Highway 10 (I-10) in Phoenix near the Phoenix airport. In the presentation, specify the approximate amount of time spent on the solution.

ASSUMPTIONS

1. Four lanes (two lanes each direction) from Flagstaff to Loop 101 in Phoenix
2. Eight lanes in Phoenix
3. Assume entire roadbed is concrete
4. Neglect on and off ramps, bridge supports and railings, and emergency stop lanes

COLLECTED DATA

1. Concrete costs $95 per cubic yard delivered to site (estimate from contractor)
2. Average depth of roadbed is 12 inches (Department of Transportation)
3. Lane width is 12 feet (Department of Transportation)
4. Unit relationships: 1 mi = 5280 ft
   1 yd = 3 ft
   1 ft = 12 in
5. Data from 2009 State Farm Road Atlas

<table>
<thead>
<tr>
<th></th>
<th>miles</th>
<th>lanes</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Flagstaff to Phoenix Loop 101</td>
<td>124</td>
<td>4</td>
</tr>
<tr>
<td>b. Loop 101</td>
<td>21</td>
<td>8</td>
</tr>
</tbody>
</table>

CALCULATIONS

Volume \( V \) = Length \( L \) x Width \( W \) x Depth \( D \)
Cost \( C \) = Volume \( V \) x (cost/cubic yard)

\[ V_s = (124 \text{ mi})(5280 \text{ ft/mi})(4)(12 \text{ ft})(12 \text{ in})(1 \text{ ft/12 in})(1 \text{ yd}^3/27 \text{ ft}^3) = 1.164(10)^8 \text{ yd}^3 \]

\[ V_y = (21 \text{ mi})(5280 \text{ ft/mi})(8)(12 \text{ ft})(12 \text{ in})(1 \text{ ft/12 in})(1 \text{ yd}^3/27 \text{ ft}^3) = 0.3942(10)^8 \text{ yd}^3 \]

\[ V = 1.558(10)^8 \text{ yd}^3 \]

\[ C = (1.558(10)^8 \text{ yd}^3)($95/\text{yd}^3) = $148,000,000 \]

Discussion/Conclusions

1. Cost of concrete may vary considerably in a short time period. $95 cost per yard used was an average of several reports found on Internet.

2. Time estimate: 40 min (obtaining data and calculations) + 30 min (write-up) = 70 min.
6.5 Solve the following problems and give the answers rounded to the proper number of significant digits.

(a) \( v = 0.0214t^2 + 0.3635t + 2.25 \) for \( t = 32 \)
(b) \( 24.56 \text{ ft} \times 12 \text{ in./ft} = ? \text{ inches} \)
(c) \( $400 \text{ a plate } \times 24 \text{ guests} = ? \)
(d) \( V = \frac{\pi(4.62 \text{ cm})^2(7.53 \text{ cm})}{3} = ? \text{ cm}^3 \) (volume of a cone)
(e) \( 325.03 + 527.897 - 615.0 = \)
(f) \( 32 \times \text{ per part } \times 45 = 250 \text{ parts} = ? \)

6.6 A pressure gauge on an air tank reads 210 pounds per square inch (psi). The face of the gauge says ±3% at 180 psi.

(a) What is the range of pressure in the tank when the gauge reads 210 psi?
(b) What is the range of air pressure in the tank when the gauge reads 87 psi?

6.7 A vacuum gauge reads 86 kPa. The face of the gauge says ±3.3 kPa at 85 kPa.

(a) What is the actual range of vacuum?
(b) What is the range when the gauge reads 130 kPa?

6.8 What is the percent of error if you use a pair of calipers on a 6-in. precision gauge block and get a reading of 5.988?

For problems 6.9 to 6.32, develop and present a solution in a manner demonstrated in Example Problems 6.2, 6.3, or 6.4. Your solution discussion should indicate the amount of time required for developing and preparing the solution. The problems are grouped into four categories: individual in-class, individual homework, team in-class, and team homework. Groups of two or three students are best for the team problems.

Individual in-class problems

6.9 Estimate the amount of time during a typical class day that you spend walking. Also estimate the number of steps you take during this time.

6.10 Estimate the number of hours that you spend watching television in a typical week during the academic semester.

6.11 Estimate the number of tennis balls that will fit in a cubic box 3 feet on a side.

6.12 Estimate the number of quarters that will fit in a box 16 inches by 10 inches by 12 inches.

6.13 Estimate the number of basketballs that will fit into your classroom. Assume room is empty of students and furniture.

6.14 Estimate the number of hours you spent on a computer during a typical week this semester. Include "surfing the net" as well as research and class requirements. Carefully document each of the categories of use.

Team in-class problems

6.15 Estimate the amount of paint required to change the color of your classroom walls.

6.16 Estimate the volume required to store 15 000 basketballs.

6.17 Estimate the number of regular M&M's needed to fill a two-liter bottle.

6.18 Estimate the volume of water used to take showers by the members of this class in one academic semester.

The following are general specifications for the Boeing 787-8 Dreamliner. Use the data to perform the estimations required in problems 6.19–6.21.

- Maximum takeoff weight: 502 500 lb (227 930 kg)
- Empty weight: 242 000 lb (109 770 kg)
- Maximum fuel capacity: 33 528 gal (126 917 L)
- Passengers: 224*
- Range: 7650–8200 nmi
- Cruise speed (Mach number): \( M = 0.85 \)

*Occupancy ranges from 210 to 260, depending upon the configuration for first, business, and tourist classes.
6.19 Estimate the weight of the crew, passengers, cargo, and fuel for a sold-out flight.

6.20 Using the information in Example Problem 4.1, estimate the flight time and fuel consumed on a flight from Tokyo to San Francisco. Assume the speed of sound at cruising altitude is 975 feet per second (fps).

6.21 Estimate the fuel consumption on a per-hour basis for a maximum range flight at an altitude where the speed of sound is 975 fps.

Individual homework problems

6.22 Estimate the number of minutes students in your engineering college spend on their cell phones in a typical academic week. A survey of a representative segment of students is necessary. Compare your results with other members of this class.

6.23 Estimate the weight, in pounds, of cars in the parking lot closest to the building where this class is held. Assume all parking spots are occupied.

6.24 Estimate the weight of water in a swimming pool on or near your campus.

6.25 Estimate the area of the running surface of an outdoor track on or near your campus.

6.26 Estimate the volume of a conical pile of sand that you have approximated the base circumference to be 210 feet. Hint: consider the angle of repose.

6.27 Estimate the weight of concrete in a 4-lane highway segment that begins as a 3% grade climb from 1000 feet of altitude to 4000 feet and then a 6% downgrade to 2500 feet.

Team homework problems (these problems will involve significant research, a specific plan for activity is strongly recommended)

6.28 Estimate the volume and cost of water used by a family of five living in a detached home during a one-year period.

6.29 Estimate the cost of concrete for a segment of interstate highway designated by your instructor. Include the rural and city components (number of lanes), interchanges, and extended width of lanes for emergency stopping.

6.30 Estimate the number of soccer balls that can be transported in a railroad box car.

6.31 Estimate how much carpet would be needed to carpet the building in which this class is held.

6.32 Your trucking company has been asked to transport a huge pile of sand from a pit to a construction site 35 miles away. The pit and construction site are both within a mile of the same two-lane paved road. The base of the sand pile covers approximately half an acre. Local laws allow only single-axle dump trucks on the highway. Your company has 14 single-axle trucks and drivers available. Estimate the time, in work days, to move the sand.